Mathematics education theories: The question of their growth, connectivity, and affinity

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Abstract. *The goal of this article is to contribute to the ongoing discussion about the meaning, growth, connectivity, and affinity of theories in mathematics education. In the first part of the article, I articulate a systemic view of theory in mathematics education. In the second part, I discuss the problem of the growth and connectivity of theories; then I introduce the idea of affinity between theories. The article rests on the idea that connecting theories and the investigation of the affinity between theories are important endeavors not only to those who are directly involved in this new disciplinary field but also to all mathematics educators. Indeed, the practice of connecting theories or the investigation of the affinities between two or more theories helps us to elucidate what theories are. For instance, to connect different research traditions, participants must make clear the ideas, principles, and assumptions of their own theoretical approaches.*

Keywords: theories in mathematics education, theoretical principles, methodology, research questions, connectivity, affinity.

Sunto. *L'obiettivo di questo articolo è quello di contribuire alla discussione in corso circa il significato, la crescita, la connessione e l'affinità delle teorie in didattica della matematica. Nella prima parte dell'articolo si fornisce una visione sistematica del concetto di teoria in didattica della matematica. Nella seconda parte si discute il problema della crescita e della connessione tra le teorie; poi si introduce l'idea di affinità tra le teorie. L'articolo si basa sull'idea che la connessione tra le teorie e la ricerca di una loro affinità siano importanti sforzi non solo per coloro che sono direttamente coinvolti in questo nuovo campo disciplinare, ma per tutti gli educatori di matematica. In effetti, la pratica di connettere teorie o di indagare le affinità tra due o più teorie ci aiuta a chiarire che cosa intendiamo per "teoria". Per esempio, per collegare diverse tradizioni di ricerca, i partecipanti devono chiarire le idee, i principi e le assunzioni dei loro approcci teorici.*

Parole chiave: teorie in didattica della matematica, principi teorici, metodologia, domande di ricerca, connettività, affinità.

Resumen. *El propósito de este artículo es contribuir a la discusión en curso sobre lo que se puede entender por desarrollo, conexión y afinidad de las teorías en educación matemática. En la primera parte del artículo se ofrece una visión sistemática del concepto de teoría en educación matemática. En la segunda parte se propone una discusión alrededor del problema del desarrollo y de la conexión entre teorías; a este punto se introduce la idea de afinidad entre estas. El artículo se basa en la idea de que la conexión entre las teorías y la búsqueda de sus afinidades es una* *actividad de gran importancia no sólo para quienes están directamente involucrados en este nuevo sector disciplinar, sino también para todos los educadores de matemática. De hecho, la práctica de conectar teorías o investigar las afinidades entre dos o más teorías nos ayuda a entender la acepción de "teoría". Por ejemplo, en el proceso de vincular diferentes tradiciones en la investigación, se requiere que los participantes hagan explícitas las ideas, los principios y las suposiciones de sus enfoques teóricos.*

Palabras clave: teorías en didáctica de la matemática, principios teóricos, metodología, preguntas de investigación, conectividad, afinidad.

1. Introduction

The past few decades have been witness to the emergence of a number of approaches to, or theories of, mathematics education—e.g., the ontosemiotic approach (Godino, Batanero, & Font, 2007), socioepistemología (Cantoral, 2013), mathematical working spaces (Kuzniak, Tanguay, & Elia, 2016), enactivism (Reid & Mgombelo, 2015), the theory of joint action (Sensevy, 2011), and inferentialism (Noorloos, Taylor, Bakker, & Derry, 2017) to mention a few only. As a result, there has been an attempt to understand the differences and similarities that may exist among theories in mathematics education. Arguably, one of the most important efforts that have been made in this context is the one that investigates whether or not two or more theories can be put into contact with one another, how, and to what extent (Bikner-Ahsbahs & Prediger, 2014). The "connectivity" or "networking" of theories and the ensuing research practice of "connecting" them depends, of course, on what we mean by a theory in mathematics education in the first place. Naturally, the question about what a theory is in mathematics education has been asked, directly or indirectly, by many math educators—for instance, Niss (1999), Sierpinska and Lerman (1996), and Sierpinska and Kilpatrick (1998).

The goal of this article is to contribute to the ongoing discussion about the meaning, growth, connectivity, and affinity of theories in mathematics education. The article rests on the idea that connecting theories and the investigation of the affinity between theories are important endeavours not only to those who are directly involved in this new disciplinary field but also to all mathematics educators. Indeed, the practice of connecting theories helps us to elucidate what theories are. For instance, to connect different research traditions, participants must make clear the ideas, principles, and assumptions of their own theoretical approaches.

The encounter with other theoretical approaches also offers participants the opportunity to recognize theoretical similarities and differences and to inquire as to what extent two or more approaches are opposed, similar, compatible, and so on.

2. Theory

I would like to start by going back to the etymology of the term theory. The word "theory" stems from the Greek verb *theōrein*, which comes from the merging of two root words, *thea* and *horaō*.

Thea (from which the term "theatre" derives) is the outward aspect in which something shows itself—what Plato called *eidos*.

The second root word in theōrein, horaō, means to look at something attentively. Thus, it follows, as Heidegger (1977) suggested, that theōrein or theory is a form of seeing, to look at something attentively and to make it reveal itself to us through the spectacle of its appearance.

As we can see, a theory in the Greek sense is a kind of contemplative act. It is something to help us make sense of something already out there, by looking at it attentively. Classifications, like the botanical ones carried out by Aristotle, were the tools with which to do that. Finding the genus and its variants was the method used to ascertain the limits of the species. But, in this line of thought, the observed objects were not forced to appear. They were there, accessible to be collected and inspected. We have to wait until the late Middle Ages and early Renaissance to find the idea that we can *force* the object to appear. That was the role of the scientific experiment.

But the idea of the scientific experiment led to a reconceptualization of the objects of investigation. That is, one was led to reflect on what was meant by a "fact" and how a fact was evident or constituted evidence of something more general.

We can distinguish at least two main trends. One in which, following the Greeks, facts are subjected to principles or universal propositions governing the theory. In an important sense, a fact illustrates a general principle. In *Posterior Analytics*, Aristotle claims that "sense perception must be concerned with particulars, whereas knowledge depends upon the recognition of the universal" (Aristotle, *Posterior Analytics*). Hence, for Aristotle and the Ancient thinkers, a fact embodies something that transcends it. By contrast, since the early 17th century, under the influence of Francis Bacon, facts were understood by some natural philosophers as theory-free particulars. As Poovey (1998) notes, some scientists argued that "one could gather data that were completely free of any theoretical component" (1998, p. xviii). With Francis Bacon, particulars gained an epistemological prestige.

The previous comments underline the idea that a theory includes assumptions about the "nature" of facts and how the facts of a theory relate to the theory's principles. In Aristotle's approach the fact refers to general principles; the fact is a particularisation of the general. In the Baconian approach, the fact generates the principle through an inductive process. In both cases, an understanding of the reality under investigation is achieved.

Of course, this is true of theories in mathematics education too. For instance, Niss (1999) contends that a theory in math education has two goals.

First, it entails a descriptive purpose aimed at increasing understanding of the phenomena studied. Second, it has a normative purpose aimed at developing instructional design. I shall come back to the second goal and focus now on the first goal—understanding.

The understanding of the phenomena under investigation can only be achieved against the background of general principles—it can be abstract principles in the Aristotelian sense or inductive principles in the Baconian sense, but it can also be something else. The understanding of the phenomena needs to be achieved against the background of general principles, for understanding, as Hegel noticed, is a form of theoretical consciousness that is beyond the fact as such. If you remain with the fact and the fact alone, without subsuming or relating it to something else, you have not yet understood.

So, a theory necessarily comprises a set of principles. Actually, it is not just a set in the sense of a bunch of items. The principles of a theory are *conceptually organized*. It is perhaps better to see them as a kind of graph, to emphasize the idea that principles are related.

Here is an example.

One principle of constructivism is the following:

Knowledge is not passively received but built up by the cognizing subject.

Here is a second principle:

The cognizing subject not only constructs her own knowledge but she does so in an autonomous way.

The second principle adds a requirement about how the building of knowledge stated in the first principle is supposed to be achieved.

But we have more than principles in a theory. A theory is a *heuristic* device used to make sense of the world (Eagleton, 1990). As such, it asks and tries to answer questions. For instance, to follow with the constructivist example, we can ask: How do children construct the concept of number?

So, in addition to principles, we have research questions. To answer them we have to produce facts that support the answers to the questions. In order to do that we still have to find the facts that will be bearers of evidence. And the meticulous way of doing that is what the *methodology* of a theory consists of. The methodology is what is going to force the realm of reality we are interested in to show up. To use Heidegger's (1977) description, the methodology is that which makes the realm of reality "reveal itself through the spectacle of its appearance." Once seen, the appearance or phenomenon is amenable to interpretation, which may result in the understanding Niss (1999) is talking about.

Drawing on what has been said, I have suggested (2008a) that a theory in math education can be considered as a triplet (P, M, Q), where P stands for the theoretical principles, M for the methodology of the theory, and Q for the research questions that a theory investigates. Q gives us an idea of the "sensitivity" of the theory.

3. The growth of theories

Naturally, a theory grows. Theories are not fixed entities; they evolve in time. There is indeed a dialectical relationship among the various components of a theory. The dialectical relationship is mediated by the *results* that a theory produces. What this means is that the three components of a theory—P, M, and Q—change as the theory produces results. In other words, the results of a theory influence its components. For instance, with the development of more and more sophisticated digital technologies, researchers are capable of producing more sophisticated facts and analyzing them in more complex manners. Digital technologies allow researchers to improve the methodology of their theories and produce new facts. These facts are then formulated, with the aid of the theories' principles, in theoretical terms, leading to new understandings of the phenomena under consideration. In turn, the fabrication or production of facts and their theoretical formulation in the manner of results allow researchers to refine more and more the theoretical principles and research questions of their theories.

Here is an example. Around 2004, in my research laboratory, we were analyzing the role of embodiment in adolescents in some generalizing tasks. We were conducting fine-grained video analyses to understand the role of gestures, mathematical signs, and language. After watching a short passage, we started noticing the role of rhythm (Radford, Bardini, & Sabena, 2007; see also Radford & Sabena, 2015). We did not anticipate rhythm as playing a subtle and profound semiotic role in mathematics cognition. Watching the video clip over and over within the possibilities of frame-to-frame analysis, we evidenced a "fact" that was theorized through the principles of the theory: We realized that rhythm was a fundamental semiotic means of knowledge objectification. *Praat* software (Boersma & Weenink, 2017) allowed us to carry out a pitch and prosodic analysis to confirm the role of rhythm. The new results required a refinement of the theoretical principles.

A more recent example has to do with the role of emotions in teaching and learning (Radford, 2015). This example is harder to pinpoint temporally, as it was part of a long process in the course of which we were continuously seeing teachers and students engage emotionally in teaching and learning. However, for emotions to emerge as a theoretical construct took a long time.

But theories also evolve by interacting with other theories. And it is here that the question of connecting theories in mathematics education comes in.

What I have said about theories is not an account of their emergence. Such an account, which is problematic on its own, should require a different approach. In the field of connecting theories what we have is two or more theories coming into contact. Although they are always changing, the theories are already there.

There are some interesting and very specific problems that arise out of the attempt to put theories in some sort of relationship.

4. Connecting theories

To investigate what happens when theories come into an explicit relationship—for instance, when a same piece of phenomenon (a video clip for example) is analyzed by two or more theories—I suggested that it might be worthy to consider theories as positioned in something that the semiotician Lotman (1990) calls a *semiosphere*.

Let me give you a spatial metaphor for Lotman's concept.

Theories inhabit the semiosphere—a multicultural, heterogeneous, and dynamically changing space of conflicting views and meaning-making processes generated by theories and their different research cultures.

It is in the semiosphere that theories live, move, and evolve. It is in the semiosphere that theories come into a relationship.

The relationship may have different goals. Prediger, Bikner-Ahsbahs, and Arzarello (2008) identified some of them in their ZDM paper. They include contrasting theories, combining them, and even ignoring other theories!

The goal of the relationship makes the theories come close to each other. How close they come depends on the goal of their dialogue. Understanding each other may not require the same proximity as when one wants to combine or synthesize them. But the kind of relationship that can exist between theories depends also on how *compatible* theories are.

Now, how can we have a sense of how far or close or compatible theories are?

A theory can be stretched so as to come close to another one. But there are limits. One interesting historical example of a relationship between theories resulted from the dialogue that North American constructivism and German interactionism carried out in the 1990s. Those theories are certainly different in many important respects, in particular in their theoretical principles as shown for instance by their different concepts of *meaning*. In constructivism, meaning is a psychological construct. In interactionism, meaning is a socio-relational or interactional notion—it is not something that is in the head but in the interaction. The different theoretical principles of those theories define the contours of what is theoretically achievable in terms of combining them. Constructivists realized that they could incorporate something that was missing in their theory: the social dimension. But this incorporation of the social, they knew very well, had to be done in a way that is *consistent* with their general theoretical principles. As we all know, in the end, the social dimension of knowing was integrated in a way that kept intact the epistemic exigencies of their postulates, such as the autonomy of the learner in the act of learning. This is why within the North American constructivism, as Simon (2012) reminds us, it is not possible to run a social and individual analysis *at the same time*. You cannot focus on, and study, the individual and the social at once, concurrently. For the North American constructivism, the social and the individual are like those quantum entities that you cannot see simultaneously.

This interesting problem is not specific to constructivism. It appears in the theory of didactic situations (Brousseau, 1997) as well. The constructs of devolution, a didactic situation, and milieu are indeed attempts at addressing the question of the social and the individual. I don't have time here to comment on the tensions that are produced in this theory by the integration of the social in the account of learning. The point that I want to make is rather that theoretical principles offer possibilities but also set limits to what can be incorporated without becoming inconsistent.

Let me come back to the general idea of linking theories. I think that most theories—perhaps all of them—are different. There is always a gap that you will find between theories if you dig deep enough. If such a gap did not exist, theories would be reducible to a single Grand Theory and mathematics education would be a tautological discourse.

Now, the fact that two theories can *be* different, that there is always a gap, is not a reason to imagine that a dialogue between them cannot be fruitful. A dialogue between theories, however, is not easy to achieve. Here are two reasons why.

The first one has to do with the polysemy or coexistence of many possible meanings for a word or phrase. *Epistemic action* or *social interaction* may have one meaning in one theory and a different meaning in another theory.

The second reason is that theories in mathematics education reflect and refract implicit and specific national-cultural "world views." They are unavoidably immersed in those symbolic systems of cultural significations that Cornelius Castoriadis (1987), Ernst Cassirer (1955), Hegel, (2001) and others have pinpointed in their investigation of the symbolic structures of society—structures from where (implicitly or explicitly) our theories draw their views of what constitutes a good student, a good teacher, a good math lesson, and so on.

5. Boundaries

As I have just suggested, theories can be put into some sort of relationship. We can always try to connect them in some way. Now, there is a limit to what can be connected.

This limit is determined by the goal of the connection, but also by the specificities of the components (P, M, Q) of the theories that are being connected. This limit has to do with the boundary of each theory under consideration.

For Lotman (1990), a boundary is one of the primary mechanisms of semiotic individuation, something that marks the limits of a first-person form ("I," "us") in opposition to non-first-person forms ("you," "them").

Drawing on this idea, I suggest calling the boundary of a theory the "edge" that a theory cannot cross without a substantial loss of its own identity. The boundary sets the “limit” of what a theory can legitimately predicate about its objects of discourse; beyond such an edge, the theory conflicts with its own principles.

Thus, the manner in which constructivism theorizes learning can be stretched to a certain point, but we cannot make it coincide with the manner in which Vygotskian approaches theorize learning. Constructivism cannot give up its idea of the learner as an autonomous, adaptive, and self-regulating agent. If it does, then it is no longer constructivism. Constructivism would have transmuted into something else.

6. Affinities

In Section 2, I noted that the principles of a theory are *conceptually organized*. They provide a theory with an interconnected conceptual kernel from where other concepts come to be related. It often happens that a theory A seems to “resonate” with another theory B. This resonance is part of a general phenomenon that I would like to term “affinity.” Affinity can occur at the level of the methodology, the theoretical principles, and/or the research questions. However, generally speaking, the meaning of an affine object O in the theory A is different from the meaning this object O may have in the theory B. The “place” of O in A and B is usually not the same—the manner in which O is understood in A and B may not coincide. In particular, it is not possible to directly import an affine object that is part of a theory into the other theory. The reason is that a theory is a *system*. The research questions are formulated in such a way that they make sense within the concepts and vocabulary of the theoretical principles; similarly, the methodology is deeply related to the theoretical principles, which do not constitute an agglomeration of theoretical claims. The systemic nature of a theory excludes a broad ranging homomorphism that would preserve meaning in general.

Here is an example. Inferentialism includes in its conceptual kernel the idea that what distinguishes us as humans is our capacity for making our thoughts explicit through language and discursive practices. Noorloos, Taylor, Bakker, and Derry explain inferentialism as follows:

> Inferentialism is a semantic theory that explains concept formation in terms of the inferences individuals make in the context of an intersubjective practice of acknowledging, attributing, and challenging one another’s commitments. For inferentialism, inferences cannot be understood apart from the norms that exist in this intersubjective practice, the game of giving and asking for reasons, with the consequence that individual reasoning cannot be understood apart from this social, norm-laden game. Inferentialism provides an alternative characterization to constructivism’s conception of social-individual interaction that replaces the latter’s emphasis on construction with a focus on the role of reasoning in learning. (Noorloos et al., 2017, para. 2)

Inferentialism comes from a contemporary branch of semantics. It focuses on how we respond to things around us, more specifically how we respond in a reasonable manner to what we say. Although in principle there are many ways in which we may reason about what we do and say, inferentialism focuses on inferences; that is, how we deduce things from other things. Language comes to play here an important role, as it is through language that, according to inferentialism, we make our claims explicit.

Inferentialism seems to have affinities with the problems of emotions as articulated within the Vygotskian tradition (see, e.g., Radford, 2015), and maybe with embodied cognition (Edwards, Radford, & Arzarello, 2009; Radford, Arzarello, Edwards, & Sabena, in press). Naturally, emotions and embodied actions can be *reasons* for something. Yet, the themes of emotion and embodied cognition first need to be coherently subsumed under the theoretical principles of inferentialism. It may be the case that the result of subsuming emotions and embodiment under the theoretical principles of inferentialism ends up in something different from the manner in which emotions and embodiment appear in some Vygotskian contemporary theories (e.g., Radford, 2008b). The resulting systemic theoretic relationship between language, emotions, and embodiment may be different and may lead to different accounts of learning and concept formation.

7. Growth and transformation

Let me return to the question of the evolution of theories that I discussed in Section 3. The existence of a hard kernel in a theory does not prevent the theory from growing. Boundaries are continuously growing and changing. And actually, one of the most interesting effects of connecting theories is that it makes theories grow.

For instance, in a previous experiment in connecting theories, reported in the 2010 PME (see Bikner-Ahsbahs, Dreyfus, Kidron, Arzarello, Radford, Artigue, & Sabena, 2010), Abstraction in Context and Interest-Dense Situation theories entered into a semiospheric relationship. As a result, some peripheral conceptual entities, that is, entities that were not organic parts of each one of these theories, ended up gaining a more central role. This was the case of the *general epistemic need* concept. This marginal entity made its entrance through the theories' interaction.

Another example: the connection of the Semiotic Bundle and Interest-Dense Situation approaches brought forward a peripheral construct, the *epistemological gap* construct.

It seems then that when two (or more) theories position themselves towards each other to enter into a semiospheric dialogue, a halo of new conceptual possibilities is formed. Potential entities appear. But they remain in the periphery of the cluster that the theories constitute. They remain "revolving around," as the etymological sense of *periphery* intimates. An effort of objectification is required to bring the peripheral entities into attention. And, in this objectifying movement, in order to accomplish the crossing of the peripheral threshold, we need something or someone else. For in the end, it turns out, as Bakhtin was suggesting, that as "every internal experience occurs on the border, it comes across another, and in this tension-filled encounter lies its entire essence." (Bakhtin, 1984, p. 287, adapted from Todorov, 1984, p. 96).

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